**Survival Analysis: Homework 1**

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1. **Exponential Density and Survival-related Functions**
   1. estimation and interpretation

**Interpretation**: The parameter represents the instantaneous hazard rate (risk) of relapse per month. A value of 0.032 per month indicates that at any given time, a patient has approximately a 3.2% risk of relapse per month, assuming the exponential distribution holds.

**Interpretation**: The parameter represents the instantaneous hazard rate (risk) of death per month. A value of 0.013 per month indicates that at any given time, a patient has approximately a 1.3% risk of death per month, assuming the exponential distribution holds.

* 1. Using this parameter to estimate quantities
     1. Mean time to relapse: months

Mean time to death: months

* + 1. Median time to relapse: months

Mean time to death: months

* + 1. ,

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* + 1. ,

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* + 1. , which is the same with in iii, demonstrating the memoryless property of the exponential distribution.
  1. Median time of relapse: 27

Median time of death: Not estimable

1. Kaplan-Meier Survival Estimate
   1. Calculate the Kaplan-Meier estimate of the survival function by hand

Method:

Result:

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* 1. Calculate the Kaplan-Meier estimate of the survival function by R

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Log-log & Linear:

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The log-log transformation maintains bounds within [0,1] because it works on the log-log scale and back-transforms, ensuring valid probabilities. The linear (plain) approach may produce invalid bounds, especially when S(t) is close to 0 or 1.

* 1. KM Plot using R

A graph of a log-log and a log-log

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* 1. Provide the estimated median survival, the estimated 25th and 75th per-centiles

25th (Q1): 22 day

50th (Median) 90 day

75th (Q3) 180 day

The KM survival estimated corresponding is the same with

A graph with blue and red lines

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* 1. Cumulative Hazard from KM

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* 1. Nelson-Aalen Estimator

Where

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* 1. Plot different cumulative hazards and it’s log

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* 1. Fleming-Harrington estimator

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Comment: The two estimators show excellent agreement (max difference ≈ 0.0767), which is expected for well-behaved survival data.

1. **Life (Actuarial) Survival Estimate**
   1. Calculate the actuarial estimate of the survival function

Where

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* 1. Calculate the estimated hazard function at the midpoint of each time interval and plot.

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A graph with red lines

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* 1. According to the plot, the hazard varies substantially, therefore, the exponential model is not appropriate because the exponential distribution assumption is not held.

1. **Non-Informative and Informative Censoring**

T₁ is defined as the time from being placed on the waiting list to receiving a liver transplant. Censoring happens when patients die during the waiting period, withdraw from the waiting list, or have not received a transplant by the end of the study. Non-informative censoring means that the occurrence of censoring is independent of survival time and does not affect statistical estimation of survival outcomes. In this case, if patients withdraw from the list for non-health-related reasons (such as relocation or choosing not to transplant), censoring may be non-informative. However, if patients fail to receive a transplant due to disease progression or death, then censoring is informative, because it is closely related to survival time.

**Censoring mechanism for :** the censoring mechanism is likely informative, because patients may be unable to receive transplantation due to changes in their medical condition.

**Censoring mechanism for “time until at least one HLA-mismatched organ becomes available”:** the censoring mechanism is also likely informative, because organ availability and patient health status may be correlated.